THERMAL AND DIFFUSIVE RELAXATION OF AN EVAPORATING DROP WITH INTERNAL HEAT LIBERATION

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The problem of nonsteady-state evaporation or growth of a radiating drop with uniformly distributed internal heat sources is considered. The Reynolds $R = ua/\nu \ll 1$ and Peclet $P_D = ua/D \ll 1$ numbers are assumed to be small (a is the radius of the drop, u the velocity of its relative motion, and ν, D, χ the coefficients of viscosity, diffusion and thermal diffusivity of the vapor-gas medium). This enables the convective transfer of vapor and heat to be neglected, and the concentration and temperature fields to be regarded as spherically symmetric [1]. In view of the fact that the density of saturated vapor is less than the density of liquid the convective flow caused by the change in radius of the drop is not taken into account [2]. It has already been shown [3,4], that for $\varkappa \ll \varkappa_r$ ($\varkappa$, $\varkappa_r$ are the coefficients of molecular and radiative thermal conductivity) there exists a bounded region $r \leqslant (1/\alpha)\sqrt{\varkappa/\varkappa_r}$ (α is the absorption coefficient for radiation in the gas), in which the effect of radiation on the temperature relaxation of the vapor-gas medium is negligible. If the condition $a \ll (1/\alpha)\sqrt{\varkappa/\varkappa_r}$ is satisfied, then the temperature at the outer boundary of this region will be practically the same as the temperature at infinity $T = T_\infty$. This means that terms in the energy equation connected with energy transferred by radiation can be neglected. It is assumed that the free path of molecules in the gas is less than the radius of the drop, and so concentration and temperature discontinuities close to the surface of the drop can be neglected [2].

1. Fundamental Equations. The diffusive and thermal relaxation of a drop with internal heat sources is considered when the drop, which has a temperature T_0, suddenly enters a medium with temperature T_∞. All quantities referring to the drop are denoted by primed symbols, and those referring to the medium by unprimed symbols. Quantities referring to the boundary have the subscript a, those referring to the fluid or vapor have the subscript 1, and those referring to the gas have the subscript 2. Symbols for total quantities have no subscripts. Thus the total number of molecules per unit volume is $n = n_1 + n_2$. Let m be the mass of a molecule, ρ the density, and v the radial component of velocity of the medium. Then

$$\rho_1 = m_1 n_1, \qquad \rho_2 = m_2 n_2$$
$$\rho = \rho_1 + \rho_2, \qquad \rho v = \rho_1 v_1 + \rho_2 v_2\,. \tag{1.1}$$

The nonsteady-state diffusion equation for small Peclet numbers has the form

$$\frac{\partial \rho_1}{\partial t} = \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right) \qquad \begin{pmatrix} D^* = m_2 n D/\rho \\ A = (m_1 n k_T + \rho_1)\, D^*/T\,. \end{pmatrix} \tag{1.2}$$

Here D is the diffusion coefficient, k_T is the thermal diffusion ratio, and T is the absolute temperature.

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In accordance with the theory of Chapman and Enskog [5] for a binary mixture at constant pressure and in the absence of external forces, the relative velocity of the components is, in fact,

$$v_1 - v_2 = -\frac{n^2}{n_1 n_2} D\left(\frac{\partial}{\partial r}\frac{n_1}{n} + \frac{k_T}{T}\frac{\partial T}{\partial r}\right). \tag{1.3}$$

If the vapor-gas mixture is treated as an ideal gas, equation (1.3) can be transformed to the form

$$v_1 - v_2 = -\frac{\rho}{\rho_2 \rho_2}\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right). \tag{1.4}$$

The equations of continuity

$$\frac{\partial \rho_1}{\partial t} + \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\rho_1 v_1 = 0, \qquad \frac{\partial \rho_2}{\partial t} + \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\rho_2 v_2 = 0 \tag{1.5}$$

and Eq. (1.3) give

$$\frac{\partial \rho_1}{\partial t} + \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\rho_1 v_2 = \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\frac{\rho}{\rho_2}\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right). \tag{1.6}$$

Further, eliminating v_1 from Eqs. (1.1) and (1.3) we obtain

$$v_2 = v + \frac{1}{\rho_2}\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right). \tag{1.7}$$

If we set (1.7) in Eq. (1.6) the diffusion equation assumes the form

$$\frac{\partial \rho_1}{\partial t} + \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\rho_1 v = \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right). \tag{1.8}$$

The convective term in Eq. (1.8) can be neglected because of the large difference in densities of the liquid and gaseous phases, and Eq. (1.2) obtained.

When the boundary condition $v_2 = \dot{a}$ for $r = a(t)$ ($\dot{a}$ is the velocity of motion of the phase boundary) is taken into account, the vapor flux density at the surface of the drop is

$$j = \rho_{1a}(v_{1a} - \dot{a}) = -\left[\frac{\rho}{\rho_2}\left(D^* \frac{\partial \rho_1}{\partial r} + A\frac{\partial T}{\partial r}\right)\right]_a. \tag{1.9}$$

The saturated vapour concentration ρ_a is a known function of temperature. For the case in which there is a small temperature differential between the surface of the drop and the temperature of the medium far from the drop this function can be approximated by the linear function

$$\rho_{1a} = \rho_{s\infty}\left[1 + \beta(T_a - T_\infty)/T_\infty\right]. \tag{1.10}$$

The time variation of temperature at the surface of the drop is determined by solving the heat problem.

Assuming that energy transport from the drop to the gas occurs by way of radiation, diffusion, thermal conductivity, and a diffusive thermal effect, we obtain the following equation for the temperature distribution in the gas:

$$\begin{gathered}\frac{\partial T}{\partial t} = \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\left(\chi^* \frac{\partial T}{\partial r} - B\frac{\partial \rho_1}{\partial r}\right)\\ B = \frac{kTD^*}{\rho c_p}\left[\frac{5}{2}\left(\frac{1}{m_2} - \frac{1}{m_1}\right) + \frac{nk_T\rho}{\rho_1\rho_2}\right]\\ \chi^* = \chi - \frac{kTA}{\rho c_p}\left[\frac{5}{2}\left(\frac{1}{m_2} - \frac{1}{m_1}\right) - \frac{nk_T\rho}{\rho_1\rho_2}\right].\end{gathered} \tag{1.11}$$

Here $\chi = \varkappa/\rho c_p$ is the coefficient of thermal diffusivity, c_p is the specific heat, and k is Boltzmann's constant.

Since the temperature and concentration differential is assumed to be relatively small, i.e.,

$$|T_a - T_\infty| \ll T_\infty, \qquad |\rho_{1a} - \rho_{1\infty}| \ll \rho_{1\infty}$$

the coefficients D*, χ*, A and B in Eqs. (1.2) and (1.11) can be treated as constants. The flux of radiative energy in the region $\alpha_r \ll \sqrt{\varkappa/\varkappa_r}$ ($\varkappa \ll \varkappa_r$) is [4]

$$S_r = \varepsilon\sigma (T_a^4 - T_\infty^4)\, a^2/r^2 \simeq 4\varepsilon\sigma T_\infty^3 (T_a - T_\infty)\, a^2/r^2 . \tag{1.12}$$

Here α is the radiation absorption coefficient in the gas, $\varkappa_r$ is the coefficient of radiative thermal conductivity, σ is the Stefan-Boltzmann constant, and ε is the effective degree of blackness of the surface of the drop surrounded by vapor. The energy flux transferred by diffusion as the result of the difference in enthalpies of the diffusing substances, is given by [5]

$$S_D = {}^5/_2 kT\, (n_1 v_1 + n_2 v_2 - nv) . \tag{1.13}$$

Equation (1.13) can be reduced to the following form with the help of equations (1.1) and (1.3) and the definition of the effective diffusion coefficient D*:

$$S_D = \frac{5}{2} kT \left(\frac{1}{m_2} - \frac{1}{m_1}\right)\left(D^* \frac{\partial \rho_1}{\partial r} + A \frac{\partial T}{\partial r}\right) . \tag{1.14}$$

The energy transfer caused by the diffusive thermal effect can be calculated from [5] the equation

$$S_t = nkTk_T (v_1 - v_2) = -\frac{nkTk_T\rho}{\rho_1\rho_2}\left(D^* \frac{\partial \rho_1}{\partial r} + A \frac{\partial T}{\partial r}\right) . \tag{1.15}$$

The energy flux transferred by thermal conductivity in a vapor-gas medium with a coefficient of thermal conductivity $\varkappa$ is

$$S_\varkappa = -\varkappa \frac{\partial T}{\partial r} . \tag{1.16}$$

The energy transport equation in a vapor-gas medium has the following form when terms associated with the mean mass flux and internal friction processes are omitted:

$$\rho c_p \frac{\partial T}{\partial t} + \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)(S_r + S_D + S_t + S_\varkappa) = 0 . \tag{1.17}$$

Remembering that, in accordance with (1.12),

$$\left(\frac{\partial}{\partial r} + \frac{2}{r}\right) S_r = 0$$

we can obtain Eq. (1.11) from (1.17).

The energy transport equation for $r < a$ is

$$\frac{\partial T'}{\partial t} = \chi' \left(\frac{\partial}{\partial r} + \frac{2}{r}\right)\frac{\partial T'}{\partial r} + \frac{q}{\rho' c_p'} . \tag{1.18}$$

Here q is the intensity of internal heat sources per unit volume of the drop. The initial and boundary conditions for the system of equations (1.2), (1.11), and (1.18) are

$$T' = T_0, \quad T = T_\infty, \quad \rho_1 = \rho_{1\infty} \quad \text{for } t = 0,\ r \neq a$$
$$T' \neq \infty \quad \text{for } r = 0, \qquad T \to T_\infty, \quad \rho_1 \to \rho_{1\infty} \quad \text{for } r \to \infty$$

$$T = T' = T_a, \quad \rho_1 = \rho_{1a} \quad \text{for } r = a, \ t \neq 0$$

$$-\varkappa\left(\frac{\partial T}{\partial r}\right)_a + 4\varepsilon\sigma T_\infty^3 (T_a - T_\infty) - \gamma\left[D^*\left(\frac{\partial \rho_1}{\partial r}\right)_a + A\left(\frac{\partial T}{\partial r}\right)_a\right] = -\varkappa'\left(\frac{\partial T'}{\partial r}\right)_a \tag{1.19}$$

$$\left(\gamma = \frac{\rho}{\rho_1} L\right).$$

Here L is the specific heat of vaporization at the temperature of the surface. The last of conditions (1.19) indicates the energy flux balance at the surface of the drop. It should be noted that an error was made in [10] in determining γ.

2. Calculation of the Temperature Distribution. A solution of Eqs. (1.2), (1.11), and (1.18) with equations (1.19) can be sought with the help of the Laplace transforms

$$F = \frac{r}{a}\int_0^\infty \frac{T - T_\infty}{T_\infty} e^{-st} dt, \quad F' = \frac{r}{a}\int_0^\infty \frac{T' - T_\infty}{T_\infty} e^{-st} dt, \quad G = \frac{r}{a}\int_0^\infty \frac{\rho_1 - \rho_{1\infty}}{\rho_{1\infty}} e^{-st} dt.$$

The Laplace transforms of the initial equations are

$$sG = D^* \frac{d^2G}{dr^2} + sA^* \frac{d^2F}{dr^2} \qquad \left(A^* = A \frac{T_\infty}{\rho_{1\infty}}\right)$$

$$sF = \chi^* \frac{d^2F}{dr^2} - sB^* \frac{d^2G}{dr^2} \qquad \left(B^* = B \frac{\rho_{1\infty}}{T_\infty}\right) \tag{2.1}$$

$$sF' = \chi' \frac{d^2F'}{dr^2} + \left(\frac{3\chi'}{sa^2} Q + \frac{T_0 - T_\infty}{T_\infty}\right)\frac{r}{a} \qquad \left(Q = \frac{qa^2}{3\varkappa' T_\infty}\right).$$

The boundary conditions become

$$F' = 0 \quad \text{for } r = 0, \qquad F \to 0, \quad G \to 0 \quad \text{for } r \to \infty$$

$$G = G_a - \frac{\rho_{s\infty} - \rho_{1\infty}}{s\rho_{1\infty}} + \beta^* F_a, \quad F' = F = F_a \quad \left(\beta^* = \beta \frac{\rho_{s\infty}}{\rho_{1\infty}}\right) \quad \text{for } r = a$$

$$\frac{1}{a}(\varkappa^* - \varkappa' - D^*\gamma^*\beta^*) F_a - \gamma^* D^* \left[\left(\frac{dG}{dr}\right)_a - \frac{G_a}{a}\right] = (\varkappa + A^*\gamma^*)\left(\frac{dF}{dr}\right)_a - \varkappa'\left(\frac{dF'}{dr}\right)_a \tag{2.2}$$

$$\left(\varkappa^* = \varkappa + 4a\varepsilon\sigma T_\infty^3 + A^*\gamma^* + \gamma^* D^*\beta^*, \quad \gamma^* = \gamma \frac{\rho_{1\infty}}{T_\infty}\right).$$

The following functions are a solution of these equations for the known boundary values $G_a F_a$:

$$G = G_1 \exp[-\mu_1 \sqrt{s}(r - a)] + G_2 \exp[-\mu_2 \sqrt{s}(r - a)]$$

$$F = F_1 \exp[-\mu_1 \sqrt{s}(r - a)] + F_2 \exp[-\mu_2 \sqrt{s}(r - a)]$$

$$F' = F_a \frac{\operatorname{sh} r\sqrt{s/\chi'}}{\operatorname{sh} a\sqrt{s/\chi'}} + \frac{1}{s}\left(\frac{3\chi' Q}{sa^2} + \frac{T_0 - T_\infty}{T_\infty}\right)\left(\frac{r}{a} - \frac{\operatorname{sh} r\sqrt{s/\chi'}}{\operatorname{sh} a\sqrt{s/\chi'}}\right)$$

$$G_1(1 + \Lambda) = G_a - \frac{\mu_2^2 A^* F_a}{1 - \mu_2^2 D^*}, \qquad G_2(1 + \Lambda) = \frac{\mu_2^2 A^* F_2}{1 - \mu_2^2 D^*} \tag{2.3}$$

$$F_1(1 + \Lambda) = -\frac{\mu_1^2 B^* G_1}{1 - \mu_1^2 \chi^*}, \qquad F_2(1 + \Lambda) = F_a + \frac{\mu_1^2 B^* G_a}{1 - \mu_1^2 \chi^*}$$

$$\mu_{1,2} = \left[\frac{\chi^* + D^* \pm \sqrt{(\chi^* - D^*)^2 - 4A^*B^*}}{2(\chi^* D^* + A^* B^*)}\right]^{1/2}, \qquad \Lambda = \frac{\mu_1^2 \mu_2^2 A^* B^*}{(1 - \mu_1^2 \chi^*)(1 - \mu_2^2 D^*)}.$$

We have, from equations (2.3),

$$\left(\frac{dG}{dr}\right)_a = -\frac{(\mu_1 + \mu_1\Lambda + \mu_2\Lambda)\sqrt{s}}{(1 + \Lambda)^2}\left(G_a - \frac{\mu_2^2 A^* F_a}{1 - \mu_2^2 D^*}\right)$$

$$\left(\frac{dF}{dr}\right)_a = -\frac{(\mu_2 + \mu_2\Lambda + \mu_1\Lambda)\sqrt{s}}{(1 + \Lambda)^2} F_a - \frac{(\mu_2 - \mu_1 + \mu_2\Lambda)\sqrt{s}\,\mu_1^2 B^*}{(1 + \Lambda)^2 (1 - \mu_1 \chi^*)} G_a \tag{2.4}$$

$$\left(\frac{dF'}{dr}\right)_a = F_a \frac{\sqrt{s}}{\sqrt{\chi'}} \operatorname{cth} \frac{a\sqrt{s}}{\sqrt{\chi'}} + \frac{1}{sa}\left(\frac{3\chi' Q}{sa^2} + \frac{T_0 - T_\infty}{T_\infty}\right)\left(1 - \frac{a\sqrt{s}}{\sqrt{\chi'}} \operatorname{cth} \frac{a\sqrt{s}}{\sqrt{\chi'}}\right).$$

Inserting (2.4) in the boundary conditions (2.2) we obtain

$$F_a = \frac{a^2}{p\chi'} \frac{M + N\sqrt{p} + [3Q/p + (T_0 - T_\infty)/T_\infty](1 - \sqrt{p}\,\mathrm{cth}\sqrt{p})}{M_* - N_*\sqrt{p} - \sqrt{p}\,\mathrm{cth}\sqrt{p}}$$

$$p = a^2 s/\chi', \quad M = \gamma^* D^* (\rho_{s\infty} - \rho_{1\infty})/\varkappa'\rho_{1\infty}, \quad M^* = 1 - \varkappa^*/\varkappa'$$

$$N = \left[\gamma^* D^* (\mu_1 + \mu_1\Lambda + \mu_2\Lambda) + (\mu_1 - \mu_2 - \mu_2\Lambda)\frac{(\varkappa + \gamma^* A^*)\mu_1^2 B^*}{1 - \mu_1^2\chi^*}\right] \times \frac{(\rho_{s\infty} - \rho_{1\infty})\sqrt{\chi'}}{(1+\Lambda)^2\varkappa'\rho_{1\infty}} \tag{2.5}$$

$$N_* = \frac{\nu^* D^* (\mu_1 + \mu_1\Lambda + \mu_2\Lambda)}{\varkappa'(1+\Lambda)^2}\left(\beta^* - \frac{\mu_2^2 A^*}{1 - \mu_2^2 D^*}\right)\sqrt{\chi'}$$
$$+ \frac{(\varkappa + A^*\gamma^*)\sqrt{\chi'}}{\varkappa'(1+\Lambda)^2}[\mu_2 + \mu_1\Lambda + \mu_2\Lambda - \beta^*\mu_1^2 B^* (\mu_1 - \mu_2 - \mu_2\Lambda)/(1 - \mu_1^2\chi^*)]$$

It follows from equations (2.5) that

$$\frac{T_a(t) - T_\infty}{T_\infty} = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \frac{M + N\sqrt{p} + [3Q/p + (T_0 - T_\infty)/T_\infty](1 - \sqrt{p}\,\mathrm{cth}\sqrt{p})}{M_* - N_*\sqrt{p} - \sqrt{p}\,\mathrm{cth}\sqrt{p}} \times \exp\left(\frac{p\chi' t}{a^2}\right)\frac{dp}{p}. \tag{2.6}$$

If we do not take into account the internal generation of heat and energy transport by radiation and diffusion, then for $\rho_2 \approx \rho$ equation (2.6) should coincide with the result [6, 7] previously obtained by another method. The corresponding equations in papers [6, 7] are, however, given in a somewhat incorrect form, since a factor 1/2 was omitted in calculating the Laplace transform of $\sqrt{t}$. Moreover, in calculating the temperature in papers [6, 7] the authors did not note that poles of the integrand are situated outside the region $|\arg p| < \pi$.

The integrand has a branch point at $p = 0$. A search for the poles of the integrand reduces to investigating the roots of the equation

$$M_* - N_*\sqrt{p} - \sqrt{p}\,\mathrm{cth}\sqrt{p} = 0, \tag{2.7}$$

If roots of this equation exist for $|p| \ll 1$ then it is sufficient to confine ourselves to the first terms of the Taylor expansion of cth $\sqrt{p}$ when searching for them:

$$p + 3N_*\sqrt{p} + 3(1 - M_*) = 0. \tag{2.8}$$

Since $1 - M_* > 0$, $N_* > 0$, the roots of this equation $\sqrt{p_1}$ and $\sqrt{p_2}$ are situated on the left-hand side of the plane of the complex variable $\sqrt{p}$ symmetrically relative to the real axis. Thus these singularities turn out to be outside the principle branch of arg p in the p plane.

In addition to these roots there exist roots for $N_* \ll 1$ whose position is determined by the following equation in the zeroth approximation:

$$M_* = \sqrt{p^{(0)}}\,\mathrm{cth}\sqrt{p^{(0)}}. \tag{2.9}$$

If $1 - M_* \ll 1$ then [8]

$$\sqrt{p_k^{(0)}} = \pm y_k i, \quad y_k = a_k - 1/a_k, \ldots, \quad a_k = (k + 1/2)\pi \quad (k = 1, 2, \ldots). \tag{2.10}$$

If corrections are sought in the following approximation $\sqrt{p_k} = \sqrt{p_k^{(0)}} + \varepsilon_k$, then we have from (2.7)

$$\varepsilon_k = N_*\sqrt{p_k^{(0)}}\,\mathrm{sh}^2\sqrt{p_k^{(0)}}/(\sqrt{p_k^{(0)}} - \mathrm{sh}\sqrt{p_k^{(0)}}\,\mathrm{ch}\sqrt{p_k^{(0)}}) \approx -N_*\sin^2 y_k. \tag{2.11}$$

It is clear from this that these poles are also situated outside the principle branch of arg p. A similar situation arises for any $N_* > 0$. In fact the roots of Eq. (2.7) for $|p| \gg 1$ are given in the zeroth approximation by

$$\sqrt{p} = \tfrac{1}{2}\,\mathrm{Ln}\,[(N_* - 1)/(N_* + 1)]. \tag{2.12}$$

It is clear that $\mathrm{Re}\sqrt{p} < 0$ for $N_* > 0$.

Thus the integrand does not have poles in the region $|\arg p| < \pi$. Calculation of the integral in (2.6) consequently reduces to calculating the integrals along the lower and upper boundaries of the branch cut made along the negative part of the real axis, and the integral around a circle of infinitely small radius enclosing $p = 0$.

Thus

$$\frac{T_a - T_\infty}{T_\infty} = \frac{Q-M}{1-M_*} + \frac{1}{\pi}\int_0^\infty \frac{dp}{\sqrt{p}}\left[N\left(M_* - \sqrt{p}\operatorname{ctg}\sqrt{p}\right) + MN_* - N_*\left(\frac{3Q}{p}\right.\right.$$
$$\left.\left. - \frac{T_0 - T_\infty}{T_\infty}\right)\left(1 - \sqrt{p}\operatorname{cth}\sqrt{p}\right)\right]\left[\left(M_* - \sqrt{p}\operatorname{cth}\sqrt{p}\right)^2 + N_*^2 p\right]^{-1}$$
$$\times \exp\left(-\frac{p\chi' t}{a^2}\right) = \frac{Q-M}{1-M_*} + \frac{1}{\pi}\int_{-\infty}^{\infty}\left[N\left(z\operatorname{ctg} z - M_*\right) - MN_*\right.$$
$$\left. + N_*\left(\frac{3Q}{z^2} - \frac{T_0 - T_\infty}{T_\infty}\right)\left(1 - z\operatorname{ctg} z\right)\right]\left[\left(z\operatorname{ctg} z - M_*\right)^2 + N_*^2 z^2\right]^{-1}\exp\left(-z^2\frac{\chi' t}{a^2}\right)dz\,. \tag{2.13}$$

Since $1 - M_* \ll 1$ for gases usually, the integrand reaches a maximum with effective width of order N_* at the points $z_1 = \pm\sqrt{3(1 - M_*) - {}^9/_4 N_*^2}$, situated in the region $|z| \ll 1$. The position of the second maximum corresponds approximately to $z_2 = \pm {}^3/_2\pi$.

If

$$\tau_0 \ll t \lesssim \tau \quad (\tau_0 = 4a^2 / 9\pi^2\chi', \quad \tau = a^2 / 3\chi' (1 - M_*))$$

then the integrand can be taken to be nonzero only in the region $|z| \lessdot z_1$, since the remaining maxima of the integrand whose positions can be determined from the equation $M_* \operatorname{tg} z = z$ in the zeroth approximation, contribute negligibly to the integral because of the presence of an exponential factor.

Thus for $N_* \ll z_1$ the integral (2.13) can be represented in the form

$$\frac{T_a - T_\infty}{T_\infty} = \frac{Q-M}{1-M_*} - \frac{1}{\pi}\left[N_*\left(\frac{3Q}{z_1^2} - \frac{T_0 - T_\infty}{T_\infty}\right)\left(1 - z_1\operatorname{ctg} z_1\right)\right.$$
$$\left. - MN_*\right]\exp\left(-\frac{t}{\tau}\right)\int_{-\infty}^{\infty}\frac{9dz}{(z^2 - z_1^2)^2 + 9N_*^2 z_1^2} = \frac{Q-M}{1-M_*} + \left(\frac{T_0 - T_\infty}{T_\infty} - \frac{Q-M}{1-M_*}\right)\exp\left(-\frac{t}{\tau}\right)\,. \tag{2.14}$$

In the absence of energy transfer by radiation and diffusion, and also when there is no thermal diffusion or diffusive thermal effect, the expression for τ coincides with that obtained previously [6, 7].

The characteristic temperature relaxation time at the surface of the drop can be estimated assuming that the change in T_a is a much slower process than the establishment of concentration and temperature fields for constant boundary conditions.*

Equation (1.18) and the boundary condition (1.19) give

$$\rho' c_p' \int_0^a \frac{r^2}{a^2}\frac{\partial T'}{\partial t}dr = \frac{q}{3} + \varkappa'\left(\frac{\partial T'}{\partial r}\right)_a = \frac{q}{3} + \gamma D^*\left(\frac{\partial \rho_1}{\partial r}\right)_a - 4\varepsilon\sigma T_\infty^3 (T_a - T_\infty) + (\varkappa + A\gamma)\left(\frac{\partial T}{\partial r}\right)_a. \tag{2.15}$$

If $t \gg a^2/\chi'$, $t \gg a^2/\chi^*$, $t \gg a^2/D^*$ we may make the following approximation in the left-hand side of Eq. (2.15):

$$\frac{\partial}{\partial t}T'(r, t) \approx \frac{d}{dt}T_a(t).$$

*Yu. S. Sedunov, Formation Kinetics of a Cloud Spectrum, Doctoral Dissertation, 1967.

The temperature and concentration fields will then be close to steady-state fields corresponding to the instantaneous values of the boundary conditions

$$\left(\frac{\partial T}{\partial r}\right)_a = -\frac{T_a - T_\infty}{a}, \quad \left(\frac{\partial \rho_1}{\partial r}\right)_a = -\frac{\rho_{s\infty} - \rho_{1\infty}}{a} - \frac{\beta\rho_{s\infty}(T_a - T_\infty)}{aT_\infty}. \tag{2.16}$$

Inserting (2.16) in Eq. (2.15) we obtain

$$\frac{a^2}{3}\rho' c_p' \frac{dT_a}{dt} = \frac{qa}{3} - \gamma D^*(\rho_{s\infty} - \rho_{1\infty}) - \left(\varkappa + 4a\varepsilon\sigma T_\infty^3 + A\gamma + \frac{\gamma\beta\rho_{s\infty} D^*}{T_\infty}\right)(T_a - T_\infty). \tag{2.17}$$

This leads to a result which corresponds with that obtained previously, i.e., $\tau = a^2/3\chi'(1 - M_*)$.

For large times when the inequalities

$$t \gg \tau, \quad t \gg \tau_\infty \; (\tau_\infty = N_*^2\tau / (1 - M_*))$$

are fulfilled, the main contribution to the integral (2.13) comes from the small neighborhood of the point $z = 0$, within the limits of which

$$z^2 \ll 3(1 - M_*), \quad N_*^2 z^2 \ll (1 - M_*)^2.$$

In calculating the integral in front of the exponential function, the asymptotic value for small z may be used to replace it:

$$\frac{T_a - T_\infty}{T_\infty} = \frac{Q - M}{1 - M_*}\left[1 - \left(\frac{N}{Q - M} + \frac{N_*}{1 - M_*}\right)\frac{a}{\sqrt{\pi\chi' t}}\right]. \tag{2.18}$$

If the condition $N_*^2 \gg 1 - M_*$ were satisfied then the quantity τ_∞ would be the characteristic temperature relaxation time at the surface of the drop. However, only the case $\tau \gg \tau_\infty$ is evidently realized.

For times $t \gg \tau$ the temperature at the surface differs little from the steady-state temperature and, as is clear from Eq. (2.17), heat losses in warming the drop can be neglected in this case.

In the absence of heat transfer by diffusion, thermal diffusion, and thermal-effect diffusion ($A = B = 0$) the diffusive and thermal fluxes in the gas for constant boundary conditions are known [9] to vary as follows:

$$\begin{aligned} j &= \frac{\rho D^*}{\rho_2 a}(\rho_{1a} - \rho_{1\infty})\left(1 + \frac{a}{\sqrt{\pi D^* t}}\right) \\ S_\varkappa &= \frac{\varkappa}{a}(T_a - T_\infty)\left(1 + \frac{a}{\sqrt{\pi\chi t}}\right). \end{aligned} \tag{2.19}$$

The heat flux transferred by molecular thermal conductivity in the drop is taken to be equal to its steady state value [10]

$$-\varkappa'\left(\frac{\partial T'}{\partial r}\right)_a = \frac{1}{3}qa. \tag{2.20}$$

The equation of balance for the energy fluxes at the boundary of the drop has the form

$$\frac{\varkappa}{a}(T_a - T_\infty)\left(1 + \frac{a}{\sqrt{\pi\chi t}}\right) + 4\varepsilon\sigma T_\infty^3(T_a - T_\infty) + \frac{\gamma D^*}{a}(\rho_{1a} - \rho_{1\infty})\left(1 + \frac{a}{\sqrt{\pi D^* t}}\right) = \frac{1}{3}qa. \tag{2.21}$$

Equation (2.21) together with the boundary condition (1.10) enables us to calculate the change in temperature at the surface of the drop for large times

$$\frac{T_a - T_\infty}{T_\infty} = \frac{\varkappa'\rho_{1\infty}Q - \gamma^* D^*(\rho_{1a} - \rho_{1\infty})}{\rho_{1\infty}(\varkappa + 4a\varepsilon\sigma T_\infty^3 + \beta^*\gamma^* D^*)}\left[1 - \left(\frac{\gamma^* D^*(\rho_{1a} - \rho_{1\infty})}{(\varkappa'\rho_{1\infty}Q - \gamma^* D^*(\rho_{1a} - \rho_{1\infty})} + \frac{\varkappa\sqrt{D^*/\chi} + \beta^*\gamma^* D^*}{\varkappa + 4a\varepsilon\sigma T_\infty^3 + \beta^*\gamma^* D^*}\right)\frac{a}{\sqrt{\pi D^* t}}\right]. \tag{2.22}$$

This equation coincides with (2.18) if we take A = B = 0 in the latter.

3. The Diffusion Flux and Heat Flux Caused by Molecular Thermal Conductivity. The Laplace transforms of that part of the diffusive flux j_0 proportional to the concentration gradient, and of the energy flux S_χ transferred by molecular thermal conductivity, have the form

$$J(s) = \int_0^\infty j_0(t) \exp(-st)\,dt, \qquad I(s) = \int_0^\infty S_\varkappa(t) \exp(-st)\,dt. \tag{3.1}$$

It follows from equations (1.9) and (1.16) that

$$\begin{gathered} J(s) = \frac{\rho D^*}{\rho_2 a}\left[(1 + \sqrt{\pi\tau_D s})\frac{\rho_{s\infty} - \rho_{1\infty}}{s} + B\rho_{s\infty}F_a + \left(B\rho_{s\infty} - \frac{\rho_{1\infty}\mu_2^2 A^*}{1 - \mu_2^2 D^*}\right)\sqrt{\pi\tau_D s}F_a\right] \\ I(s) = \frac{\varkappa T_\infty}{a}\left[(1 + \sqrt{\pi\tau_\chi s} + \beta^*\sqrt{\pi\tau^* s})F_a + \frac{(\rho_{s\infty} - \rho_{1\infty})\sqrt{\pi\tau^*}}{\rho_{1\infty}\sqrt{s}}\right] \\ \tau_D = \frac{a^2(\mu_1 + \mu_2\Lambda + \mu_1\Lambda)^2}{\pi(1+\Lambda)^4}, \qquad \tau_\chi = \frac{a^2(\mu_2 + \mu_2\Lambda + \mu_1\Lambda)^2}{\pi(1+\Lambda)^4} \\ \sqrt{\tau^*} = a(\mu_2 - \mu_1 + \mu_2\Lambda)\mu_1^2 B^* / \sqrt{\pi}(1+\Lambda)^2(1 - \mu_1^2\chi^*). \end{gathered} \tag{3.2}$$

In order to find the inverse transforms of equations (3.2) we must calculate

$$\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \sqrt{s}F_a(s)\exp(st)\,ds = \frac{\sqrt{\chi'}}{\pi a}\int_{-\infty}^{\infty} \frac{\{M - [3Q/z^2 - (T_0 - T_\infty)/T_\infty]\}(M_* - z\operatorname{ctg} z) - NN_* z^2}{(M_* - z\operatorname{ctg} z)^2 + N_*^2 z^2}\exp\left(-\frac{z^2\chi' t}{a^2}\right)dz. \tag{3.3}$$

For times in the interval $\tau_0 \ll t \leqslant \tau$

$$\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \sqrt{s}F_a(s)\exp(st)\,dt = -\frac{3N_1\sqrt{\chi'}}{a}\exp\left(-\frac{t}{\tau}\right). \tag{3.4}$$

For large times $t \gg \tau$

$$\frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \sqrt{s}F_a(s)\exp(st)\,dt = \frac{Q - M}{1 - M_*}\frac{1}{\sqrt{\pi t}}. \tag{3.5}$$

Thus for times $\tau_0 \leq t \leqslant \tau$ on condition that $N_*^2 \ll 1 - M_* \ll 1$

$$j_0(t) = \frac{\rho D^*}{\rho_2 a}\left[(\rho_{s\infty} - \rho_{1\infty})\left(1 + \frac{\sqrt{\tau_D}}{\sqrt{t}}\right) + \beta\rho_{s\infty}\frac{T_a(t) - T_\infty}{T_\infty} - \frac{3N}{a}\sqrt{\chi'\pi\tau_D}\left(\beta\rho_{s\infty} - \frac{\rho_{1\infty}\mu_2^2 A^*}{1 - \mu_2^2 D^*}\right)\exp\left(-\frac{t}{\tau}\right)\right]. \tag{3.6}$$

A quasi-steady-state evaporation regime, i.e., a regime for which the density of the diffusive flux is determined by the instantaneous value of the temperature at the surface of the drop, clearly exists, from (3.6), if the following inequalities are satisfies simultaneously:

$$\begin{gathered} t \gg \tau_D, \quad \left|\frac{T_0 - T_c}{T_\infty}\right| \gg \left|\frac{3N\sqrt{\pi\chi'\tau_D}}{a\beta\rho_{s\infty}}\left(\beta\rho_{s\infty} - \frac{\rho_{1\infty}\mu_2^2 A^*}{1 - \mu_2^2 D^*}\right)\right| \\ T_c = T_\infty[1 + (Q - M)/(1 - M_*)]. \end{gathered} \tag{3.7}$$

Here T_c is the temperature at the drop surface for a steady-state evaporation regime.

If A = B = 0 then the condition for a steady-state evaporation regime (3.6) to exist for times $\tau_0 \ll t \leqslant \tau$ can be transformed to the form

$$\rho' c_p' \,|T_0 - T_c| \gg 3L\,|\rho_{s\infty} - \rho_{1\infty}|\,(\rho/\rho_2) \tag{3.8}$$

Thus when the condition $N_*^2 \ll 1 - M_* \ll 1$ is fulfilled a quasi-steady-state evaporation regime exists for the drop in the time interval $\tau_0 \ll t \leqslant \tau$ if the quantity of heat absorbed or liberated per unit volume of the drop in its temperature relaxation process is considerably in excess of the heat losses per unit volume of the vapor-gas medium, associated with the phase transition when the vapor concentration changes by an amount $|\rho_{s\infty} - \rho_{1\infty}|$.

The result of (3.8) is absent from [6], where the study of the quasi-steady-state regime was restricted to examination of whether the condition $N_*^2 \ll 1 - M_*$ was met.

When the conditions $N_*^2 \ll 1 - M_* \ll 1$ are satisfied the heat flux for times $\tau_0 \ll t \leqslant \tau$ is

$$S_\varkappa = \frac{\varkappa T_\infty}{a}\left[\frac{T_a(t) - T_\infty}{T_\infty} + \frac{(\rho_{s\infty} - \rho_{1\infty})\sqrt{\tau^*}}{\rho_{1\infty}\sqrt{t}} - \frac{3N}{a}\sqrt{\pi\chi'}\,(\sqrt{\tau_\varkappa} + \beta^*\sqrt{\tau^*})\exp\left(-\frac{t}{\tau}\right)\right]. \tag{3.9}$$

It is thus clear that a quasi-steady-state regime exists for the heat flux if

$$t \gg \tau^*, \quad \left|\frac{T_0 - T_c}{T_\infty}\right| \gg \left|\frac{3N}{a}\sqrt{\pi\chi'}\,(\sqrt{\tau_\varkappa} + \beta^*\sqrt{\tau^*})\right|. \tag{3.10}$$

For the particular case A = B = 0, (3.10) assumes the form

$$\rho' c_p' \,|T_0 - T_c| \gg 3L\sqrt{D^*/\chi}\,|\rho_{s\infty} - \rho_{1\infty}|\,(\rho/\rho_2). \tag{3.11}$$

It follows from equations (3.8) and (3.11) that the conditions for a quasi-steady-state regime to exist for the diffusive and heat fluxes are in fact the same.

Changes in the diffusive and heat fluxes for large times $t \gg \tau$ are described by the equations

$$j_0(t) = \frac{\rho D^*}{\rho_2 a}\left[(\rho_{s\infty} - \rho_{1\infty})\left(1 + \frac{\sqrt{\tau_D}}{\sqrt{t}}\right) + \beta\rho_{s\infty}\frac{T_a(t) - T_\infty}{T_\infty} + \left(\beta\rho_{s\infty} - \frac{\rho_{1\infty}\mu_2^2 A^*}{1 - \mu_2^2 D^*}\right)\frac{(Q - M)\sqrt{\tau_D}}{(1 - M_*)\sqrt{t}}\right]$$

$$S_\varkappa(t) = \frac{\varkappa T_\infty}{a}\left[\frac{T_a(t) - T_\infty}{T_\infty} + \frac{(\rho_{s\infty} - \rho_{1\infty})\sqrt{\tau^*}}{\rho_{1\infty}\sqrt{t}} + \frac{(Q - M)(\sqrt{\tau_\varkappa} + \beta^*\sqrt{\tau^*})}{(1 - M_*)\sqrt{t}}\right]. \tag{3.12}$$

Let the conditions A = B = 0 be fulfilled. It is clear from (3.12) that any change in the diffusive and heat fluxes occurs with characteristic times τ_D and τ_χ.

When conditions $\tau \gg \tau_D$, $\tau \gg \tau_\chi$, $t \gg \tau$ are fulfilled we can expect that the diffusive and heat fluxes will be practically the same as their steady-state values.

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